Name solutions

October 27, 2009

ECE 311 | Exam 2 | Fall 2009

Closed Text and Notes

1) Be sure you have 15 pages.

2) Write only on the question sheets. Show all your work. If you need more room for a particular problem, use the reverse side of the same page.

3) calculators allowed

4) Write neatly, if your writing is illegible then print.

5) The last 2 pages contain equations that may be of use to you.

6) This exam is worth 100 points.

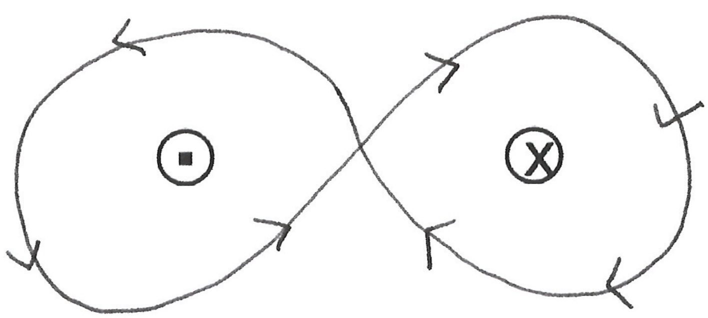
(5 pts) 1. A stream of electrons is flowing in free space all with the velocity $\mathbf{v} = 100\frac{\text{m}}{\text{s}}\mathbf{a}_x$. If the density of the electrons is $10^{20}\ \frac{\text{electrons}}{\text{m}^3}$, what is the current density?

$$\vec{J} = \rho_v \vec{v} = \left(-1.6\times10^{-19}\ \frac{C}{elec}\right)\left(10^{20}\ \frac{elec}{m^3}\right)\left(100\ \frac{m}{s}\ \hat{a}_x\right)$$

$$= -1.6\times10^{3}\ \frac{C}{m^2 s}\ \hat{a}_x$$

$$= -1.6\times10^{3}\ \frac{A}{m^2}\ \hat{a}_x$$

(5 pts) 2. Two wires are perpendicular to the page as shown. The wire on the left has a current of 1 A flowing out of the page and the one on the right a current of 1 A into the page. What is the value of $\oint \mathbf{H}\cdot\mathbf{dl}$ for the path shown?



$$\oint \vec{H}\cdot\vec{dl} = I_{enclosed} = 2\text{ A}$$

(5 pts) 3. An electric field causes a polarization field in a dielectric of $10^{-8}\ \frac{C}{m^2}$. If there are $10^{20}\ \frac{\text{atoms}}{m^3}$ in the dielectric, what are the dipole moments of the atoms?

$$P = 10^{-8}\ \frac{C}{m^2} = \frac{\text{dipole moment}}{\text{unit volume}}$$

$$10^{-8}\ \frac{C}{m^2} = \left(10^{20}\ \frac{\text{atoms}}{m^3}\right) p$$

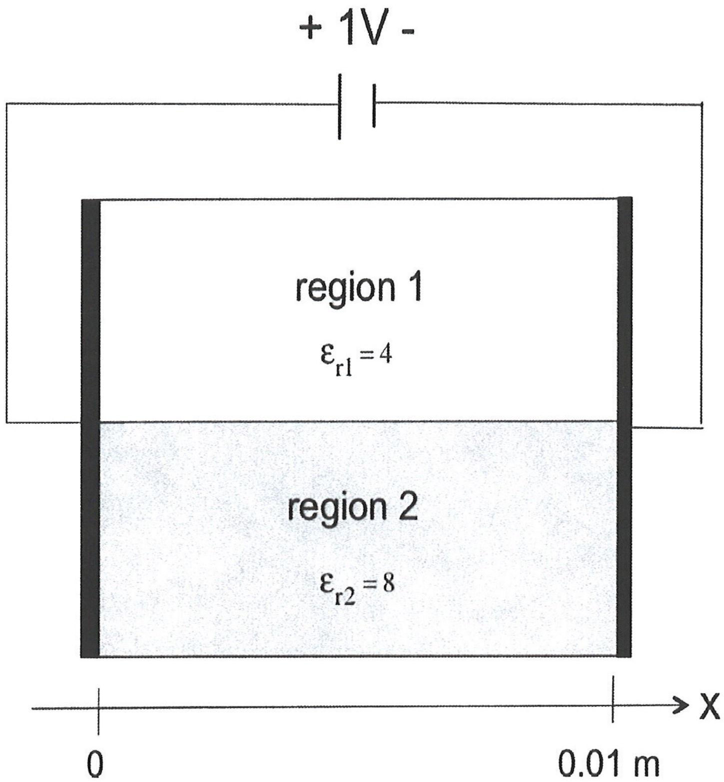
$$p = \frac{10^{-8}\ \frac{C}{m^2}}{10^{20}\ \frac{\text{atoms}}{m^3}}$$

$$p = 10^{-28}\ \frac{C\,m}{\text{atom}} = \begin{array}{c}\text{dipole moment}\\ \text{per atom}\end{array}$$

(5 pts) 4. A conducting ring of radius 1 m is in the xy-plane and centered at the origin. A current of 1 A is flowing in the $\mathbf{a}_\phi$ direction around the ring. What is the value of $\oint \mathbf{B} \cdot d\mathbf{S}$ over a spherical surface of radius 2 m centered at the origin?

$$\oint \vec{B} \cdot \vec{dS} = 0$$

(12 pts) 5. Two parallel plates of area 10^{-3} cm^2 have a potential difference of 1V and two different dielectric materials as shown. Determine **E**, **D**, and **P** in the two dielectric regions.



The electric field will be the same in the two dielectrics

$$\vec{E}_1 = \vec{E}_2 = \frac{1V}{0.01m}\hat{a}_x$$

$$\boxed{E_1 = E_2 = 100\frac{V}{m}\hat{a}_x}$$

$$\vec{D}_1 = \epsilon_0 \epsilon_{r_1} \vec{E}_1 = (8.854\times10^{-12}\frac{F}{m})(4)(100\frac{V}{m}\hat{a}_x)$$

$$\boxed{\vec{D}_1 = 3.54\times10^{-9}\frac{C}{m^2}\hat{a}_x}$$

$$\vec{D}_2 = \epsilon_0 \epsilon_{r_2} \vec{E}_2 = (8.854\times10^{-12}\frac{F}{m})(8)(100\frac{V}{m}\hat{a}_x)$$

$$\boxed{\vec{D}_2 = 7.08\times10^{-9}\frac{C}{m^2}\hat{a}_x}$$

$$\vec{P}_1 = \vec{D}_1 - \epsilon_0\vec{E}_1 = \left[3.54\times10^{-9}\frac{C}{m^2} - (8.854\times10^{-12}\frac{F}{m})(100\frac{V}{m})\right]\hat{a}_x$$

$$\boxed{\vec{P}_1 = 2.656\times10^{-9}\frac{C}{m^2}\hat{a}_x}$$

$$\vec{P}_2 = \vec{D}_2 - \epsilon_0\vec{E}_2 = \left[7.08\times10^{-9}\frac{C}{m^2} - (8.854\times10^{-12}\frac{F}{m})(100\frac{V}{m})\right]\hat{a}_x$$

$$\boxed{\vec{P}_2 = 6.198\times10^{-9}\frac{C}{m^2}\hat{a}_x}$$

(10 pts) 6. Determine a numerical value for the capacitance of a solid metal sphere of diameter 10 m. Assume everywhere else is free space and $V(r=\infty) = 0$ V.

Place a charge of $+Q$ on the sphere

The resulting electric field is

$$\vec{E} = \frac{Q}{4\pi\epsilon_0 r^2}\hat{a}_r \quad \text{for} \quad r > 5\text{ m}$$

$$= 0 \quad \text{for} \quad r < 5\text{ m}$$

$$V(10\text{m}) - V(\infty) = V(10\text{m}) = -\int_{\infty}^{5\text{m}} \frac{Q}{4\pi\epsilon_0 r^2}\hat{a}_r \cdot dr\,\hat{a}_r$$

$$V(10\text{m}) = -\frac{Q}{4\pi\epsilon_0}\int_{\infty}^{5\text{m}} \frac{dr}{r^2} = \frac{Q}{4\pi\epsilon_0 r}\Big|_{\infty}^{5\text{m}}$$

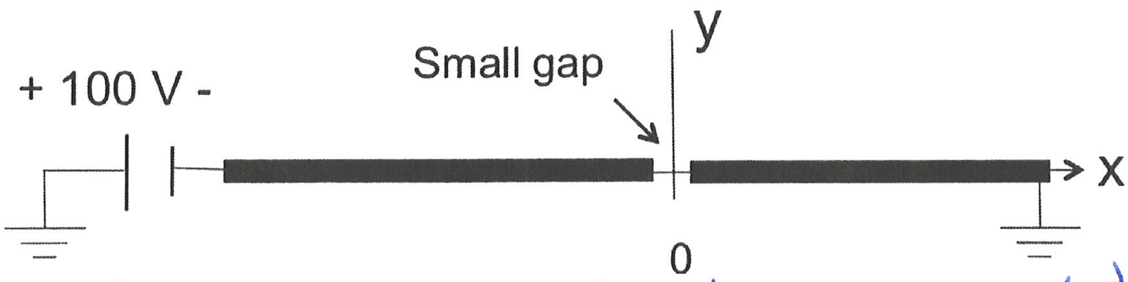
$$V(10\text{m}) = \frac{Q}{4\pi\epsilon_0 (5\text{m})}$$

$$C = \frac{Q}{V} = \frac{Q}{\left(\frac{Q}{4\pi\epsilon_0 (5\text{m})}\right)} = 4\pi\epsilon_0 (5\text{m})$$

$$= 4\pi\left(8.854\times10^{-12}\,\frac{\text{F}}{\text{m}}\right)(5\text{m})$$

$$\boxed{C = 5.56\times10^{-10}\text{ F}}$$

(15 pts) 7. The xz plane contains two semi-infinite conduction planes with a small gap along the z-axis as shown. Find the potential everywhere. (Hint you have to solve for two separate regions, $0 < \varphi < \pi$ and $\pi < \varphi < 2\pi$.)



This configuration has cylindrical symmetry with $\frac{\partial V}{\partial \rho} = \frac{\partial V}{\partial z} = 0$

$$\nabla^2 V = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\frac{\partial V}{\partial \rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2} = \frac{1}{\rho^2}\frac{\partial^2 V}{\partial \phi^2} = 0$$

$$\frac{\partial^2 V}{\partial \phi^2} = 0 \xrightarrow{\text{solution}} V(\phi) = A\phi + B$$

for $0 < \phi < \pi$ boundary conditions $V(0) = 0$ and $V(\pi) = -100V$

$V(0) = 0 = A(0) + B \Rightarrow B = 0$

$V(\pi) = -100V = A(\pi) + B \Rightarrow A = -\frac{100V}{\pi}$

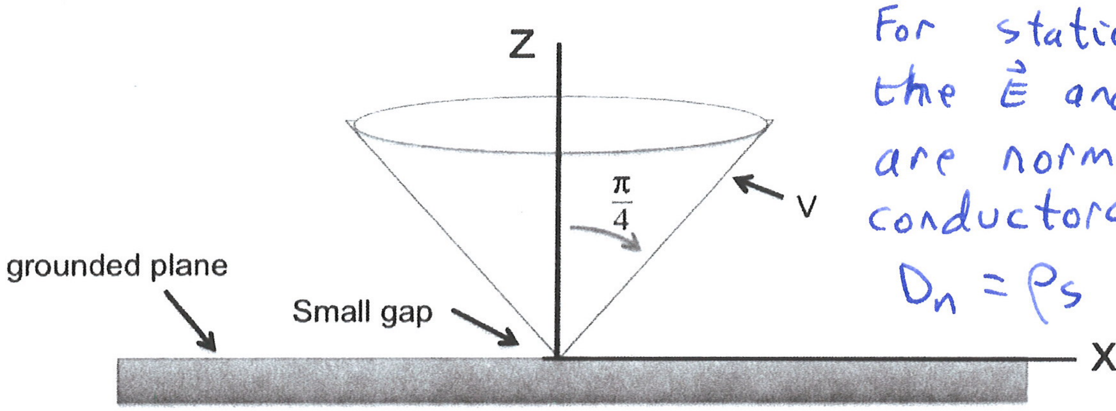
$$\boxed{V(\phi) = -\frac{100V}{\pi}\phi \quad \text{for} \quad 0 < \phi < \pi}$$

for $\pi < \phi < 2\pi$ boundary conditions $V(\pi) = -100V$ $V(2\pi) = 0$

$$\left.\begin{aligned} V(2\pi) &= A(2\pi) + B = 0 \\ V(\pi) &= A(\pi) + B = -100V \end{aligned}\right\} \quad A = \frac{100V}{\pi} \quad B = -200V$$

$$\boxed{V(\phi) = \frac{100V}{\pi}\phi - 200V \quad \text{for} \quad \pi < \phi < 2\pi}$$

(10 pts) 8. For the two conductor system shown consisting of an infinite conducting cone of half-angle $\frac{\pi}{4}$ radians and an infinite conducting plane in the xy plane, the electric field is $\mathbf{E}(r,\theta,\phi)=\frac{100V}{r\sin(\theta)}\hat{\mathbf{a}}_\theta$ for $\frac{\pi}{4}<\theta<\frac{\pi}{2}$. Find the charge density at the points $A(r=1m,\ \theta=\frac{\pi}{4},\phi=\frac{\pi}{2})$ and $B(r=1m,\ \theta=\frac{\pi}{2},\phi=\frac{\pi}{2})$. I want numerical values.



For static conditions the $\vec{E}$ and $\vec{D}$ fields are normal to the conductors and $D_n = \rho_s$

Point A is on the cone

$$\rho_s\left(1,\frac{\pi}{4},\frac{\pi}{2}\right) = D_n\left(1,\frac{\pi}{4},\frac{\pi}{2}\right) = \epsilon_0 E\left(1,\frac{\pi}{4},\frac{\pi}{2}\right)$$

$$= \left(8.854\times10^{-12}\frac{F}{m}\right)\frac{100V}{(1m)\sin\left(\frac{\pi}{4}\right)}$$

$$= 1.25\times10^{-9}\frac{C}{m^2}$$

Point B is on the plane. The normal to this plane is in the opposite direction to $\vec{E}$.

$$\rho_s\left(1,\frac{\pi}{2},\frac{\pi}{2}\right) = -D_n\left(1,\frac{\pi}{2},\frac{\pi}{2}\right) = -\epsilon_0 E\left(1,\frac{\pi}{2},\frac{\pi}{2}\right)$$

$$= -\left(8.854\times10^{-12}\frac{F}{m}\right)\frac{100V}{(1m)\sin\left(\frac{\pi}{2}\right)}$$

$$= -8.854\times10^{-10}\frac{C}{m^2}$$

(15 pts) 9. For the following current density, find **H** everywhere.

$$\mathbf{J} = 2\frac{\text{A}}{\text{m}^2}\hat{\mathbf{a}}_z \,, \text{ for } 0<\rho<1\text{m}$$
$$0 \quad , \text{ for } \quad \rho > 1\text{m}$$

From the symmetry of $\vec{J}$, $\vec{H}$ will only have a component in the $\hat{a}_\phi$ direction.

$0 < \rho < 1\text{m}$

$$\oint \vec{H} \cdot \vec{dl} = \int_0^{2\pi} H_\phi \hat{a}_\phi \cdot \rho \, d\phi \, \hat{a}_\phi = I_{encl} = \left(2\frac{A}{m^2}\right)\pi\rho^2$$

$$\rho H_\phi \int_0^{2\pi} d\phi = 2\pi\rho H_\phi = 2\pi \frac{A}{m^2}\rho^2$$

$$H_\phi = \rho \text{ (in m)}$$

$$\boxed{\vec{H} = \rho \, \hat{a}_\phi \, \frac{A}{m} \qquad 0 < \rho < 1\text{m}}$$

$\rho > 1\text{m}$

$$\oint \vec{H} \cdot \vec{dl} = \int_0^{2\pi} H_\phi \hat{a}_\phi \cdot \rho \, d\phi \, \hat{a}_\phi = I_{encl} = \left(2\frac{A}{m^2}\right)\pi(1\text{m})^2$$

$$\rho H_\phi \int_0^{2\pi} d\phi = 2\pi\rho H_\phi = 2\pi \text{ A}$$

$$H_\phi = \frac{1}{\rho}$$

$$\boxed{\vec{H} = \frac{1}{\rho} \hat{a}_\phi \, \frac{A}{m} \qquad \rho > 1\text{m}}$$

Some of you tried to use $\nabla \times \vec{H} = \vec{J}$ to find $\vec{H}$. This approach would not work for $\rho > 1\text{m}$, but it would work for $0 < \rho < 1\text{m}$

In cylindrical coordinates

$$\nabla \times \vec{H} = \left[\frac{1}{\rho}\frac{\partial H_z}{\partial \phi} - \frac{\partial H_\phi}{\partial z}\right]\hat{a}_\rho + \left[\frac{\partial H_\rho}{\partial z} - \frac{\partial H_z}{\partial \rho}\right]\hat{a}_\phi + \frac{1}{\rho}\left[\frac{\partial(\rho H_\phi)}{\partial \rho} - \frac{\partial H_\rho}{\partial \phi}\right]\hat{a}_z = 2\frac{A}{m^2}\hat{a}_z$$

From the symmetry of the problem

$$\vec{H} = H_\phi(\rho)\,\hat{a}_\phi$$

so,

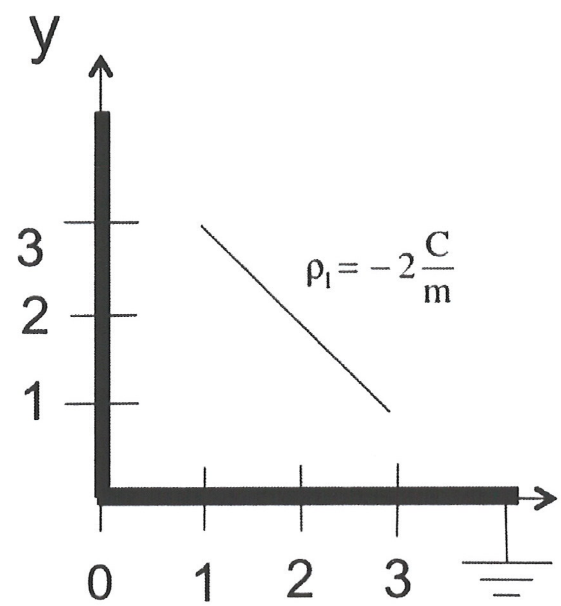
$$\frac{1}{\rho}\frac{\partial(\rho H_\phi)}{\partial \rho} = 2$$

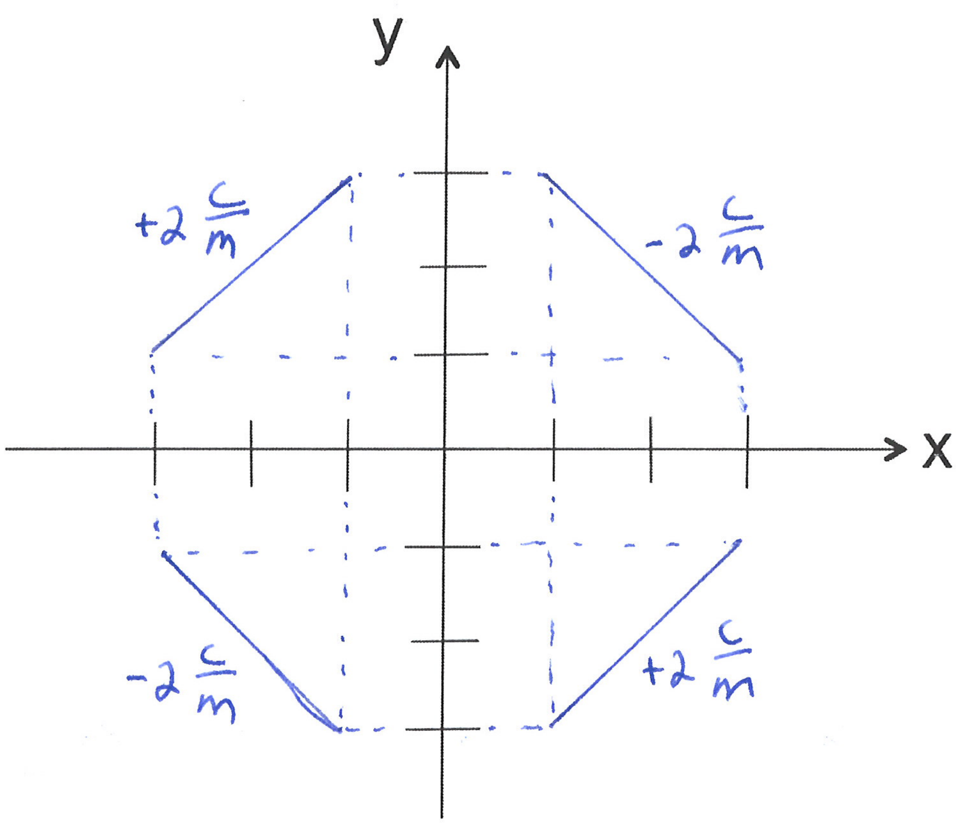
$$\frac{\partial(\rho H_\phi)}{\partial \rho} = 2\rho$$

$$\rho H_\phi = \rho^2$$

$$H_\phi = \rho \implies \vec{H} = \rho\,\hat{a}_\phi \text{ for } 0 < \rho < 1\text{m}$$

(12 pts) 10. For the grounded conductor and line charge shown, determine the image charge configuration that you could use to determine the electric field in the first quadrant.





(6 pts) 11. Fill in the table with the standard units for the following

magnetic flux density, **B**	$\frac{Wb}{m^2} = T$
Magnetic field intensity, **H**	A/m
Electric Field Intensity, **E**	V/m
Electric Flux Density, **D**	C/m^2
Electric flux, **Ψ**	C
Magnetic flux, **Ψ**	Wb